THERMAL NOISE MEASUREMENTS

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NIST/ARFTG Short Course, Broomfield, CO, November 2006

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      - simple, idealized case
      - not so simple case
    - Uncertainties
      - simple case
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- Outline (cont’d)
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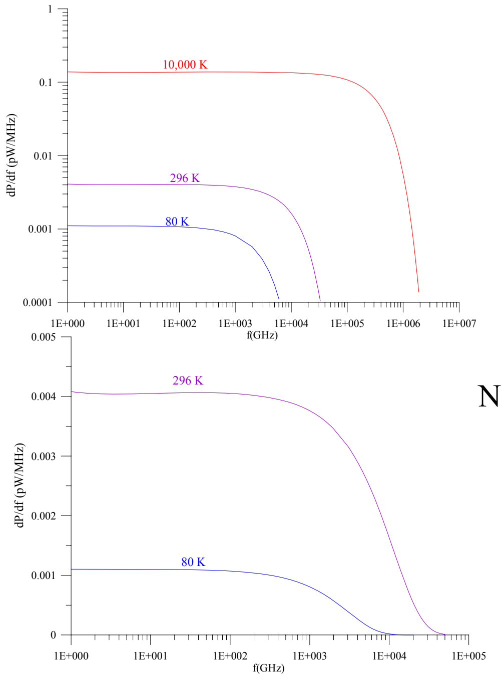
I. BASICS

Nyquist Theorem

- Derivation:
  - Electr. Eng. [1-4]
  - Physics, Stat. Mech. [4]
- For passive device, at physical temperature T, with small *Δf*,

$$\langle P_{avail}(f)\rangle = \frac{hf}{e^{hf/(k_B T)}-1}\Delta f$$



$$\left\langle P_{avail}(f)\right\rangle = \frac{hf}{e^{hf/(k_B T)}-1}\Delta f$$

Note: <u>very</u> small powers.

- Limits
  - small f: $<P_{avail}> \approx k_B T\,\Delta f\,[1 - hf/(2k_B T)]$
    $\approx k_B T\,\Delta f$
  - large f: $\rightarrow 0$
  - knee occurs around f(GHz) ≈ 20 T(K)
- Quantum effect
  - $h/k_B = 0.04799$ K/GHz
  - So at 290 K, 1 % effect at 116 GHz
    at 100 K, 1 % effect at 40 GHz
    at 100 K, 0.1 % effect at 4 GHz
    30 K @ 40 GHz → 6.4%, 0.26 dB

NOISE TEMPERATURE

- What about active devices? Can we define a noise temperature?
- Several different definitions used:
  - delivered vs. available power
  - with or without quantum effect
    i.e., does $T_{noise} \propto P_{avail}$ ("power" definition), or is T_{noise} the physical temperature that would result in that value of P_{avail} ("equivalent-physical-temperature" definition)?

- For passive case:

$$\langle P_{avail}(f)\rangle = \frac{hf}{e^{(hf/k\underset{\uparrow}{T})}-1}\Delta f \qquad \text{(Nyquist with quantum)}$$

$$\text{Small } hf/kT \Rightarrow \langle P_{avail}(f)\rangle \approx k\underset{\uparrow}{T}\Delta f$$

- Which do we preserve in defining T_{noise} for general (passive & non-passive case)?

- IEEE [5]: "(1)(general)(at a pair of terminals and at a specifice frequency) the temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual terminals."
  and
  "(4)(at a port and at a selected frequency) A temperature given by the exchangeable noise-power density divided by Boltzmann's constant, at a given port and at a stated frequency."

- We (I) will use second definition, noise temp ≡ available spectral noise-power divided by Boltzmann's constant.
- It is the common choice in international comparisons [6] and elsewhere [7].
- It is much more convenient for amplifier noise considerations, at least for careful ones. (See discussion below, under Noise Figure and Parameters.)

- So $P_{avail} \equiv k_B T_{noise} \Delta f$
- And for passive devices,

$$T_{noise} = \frac{1}{k_B}\left[\frac{hf}{e^{hf/(k_B T)} - 1}\right] \approx T_{phys}$$

- Convenient to define “Excess noise ratio”

$$ENR_{avail}(dB) \equiv 10 \log_{10}\left(\frac{T_{avail} - T_0}{T_0}\right) \qquad T_0 = 290\ \mathrm{K}$$

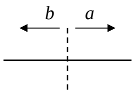
T=9500 K ⇒ ENR ≈ 15.02 dB

T=1000 K ⇒ ENR ≈ 3.89 dB

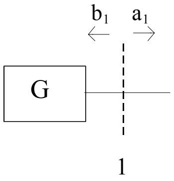
No matter what definition of noise temperature you choose, it is helpful to state your choice.

MICROWAVE NETWORKS & NOISE [8,9]

- Assume lossless lines, single mode.
- Travelling-wave amplitudes *a*, *b*.
- Normalized such that $P_{del} = |a|^2 - |b|^2$ is spectral power density.

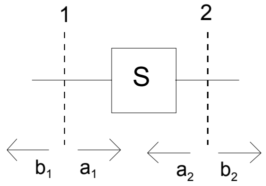


- Describe (linear) one-ports by



$$a_1 = \Gamma_G b_1 + \hat{a}_G$$

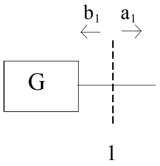
- And (linear) two-ports by



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

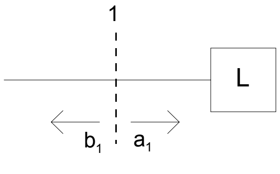
- Available power:

$$P_G^{avail} = \frac{|\hat{a}_G|^2}{1-|\Gamma_G|^2}$$

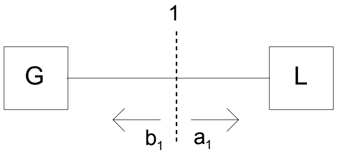


Relation to noise temp: $\left\langle |\hat{a}_G|^2 \right\rangle = \left(1-|\Gamma_G|^2\right) k_B T_G$

- Delivered power:

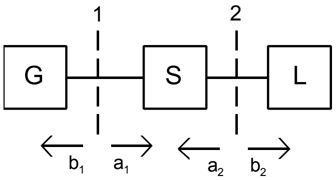


$$P_1^{del} = |a_1|^2 - |b_1|^2 = |a_1|^2 \left(1-|\Gamma_L|^2\right)$$



Mismatch Factor

$$M_1 = \frac{P^{del}}{P^{avail}} = \frac{\left(1-|\Gamma_L|^2\right)\left(1-|\Gamma_G|^2\right)}{|1-\Gamma_L \Gamma_G|^2}$$

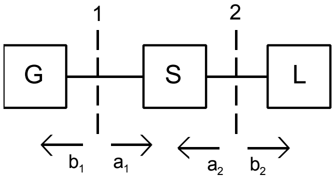


Efficiency

$$\eta_{21} = \frac{P_2^{del}}{P_1^{del}} = \frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_L S_{22}|^2\left(1-|\Gamma_{SL}|^2\right)}$$

$$= \frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_L S_{22}|^2 - |(S_{12}S_{21}-S_{11}S_{22})\Gamma_L + S_{11}|^2}$$

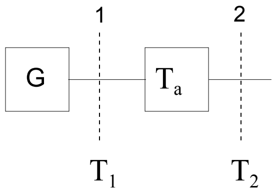
- Available power ratio (available gain):
  $\alpha_{21} \equiv p_{2,avail}/p_{1,avail}\ (\hat{b}_1\ ,\ \hat{b}_2 = 0)$



$$\alpha_{21} = \frac{|S_{21}|^2\left(1-|\Gamma_G|^2\right)}{|1-\Gamma_G S_{11}|^2\left(1-|\Gamma_{GS}|^2\right)}$$

$$\Gamma_{GS} = S_{22} + \frac{S_{12}S_{21}\Gamma_G}{1-\Gamma_G S_{11}}$$

- Temperature translation through a passive, linear, 2-port (attenuator, adapter, line, ...)



$$P_2^{avail} = \alpha_{21} P_1^{avail} + f_0(T_a)$$
$$T_2 = \alpha_{21} T_1 + f(T_a)$$

Say $T_1 = T_a$, then T_2 must $= T_a$, so

$$T_2 = T_a = \alpha_{21} T_a + f(T_a)$$
$$f(T_a) = (1 - \alpha_{21}) T_a$$

and therefore

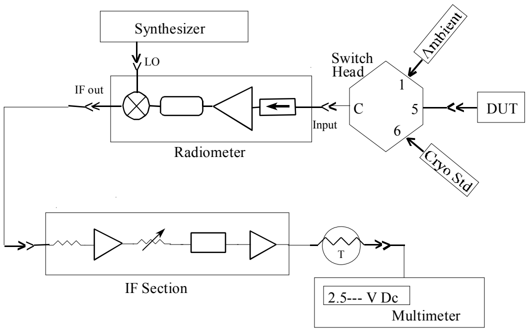
$$T_2 = \alpha_{21} T_1 + (1 - \alpha_{21}) T_a$$

II. NOISE-TEMPERATURE MEASUREMENT

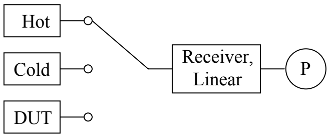
Total-Power Radiometer [10-12]

- Radiometer: measures “radiated” power. For us, measures delivered power (in w.g. or transmission line), & we convert to available power & therefore to noise temperature.
- Two principal types of radiometer for noise-temperature measurements are Dicke radiometer and total-power radiometer [10].
- Total-power radiometer is most common for lab use, & that’s what we’ll discuss.

- NIST Coaxial Radiometer, General Features:
  - Total-power radiometer, isolated (60 dB), baseband IF, double sideband, 5 MHz BW, thermistor detector.



- Simple case: symmetric, matched (all Γ's = 0)



Matched → $p_{del} = p_{avail}$ Linear → $P = a + bp_{del} = a + bp_{avail}$

2 standards (h,c) determine a, b:

$$P_h = a + bk_B BT_h$$
$$P_c = a + bk_B BT_c$$

Note: This could be written as

$$T_{out,rec} = G_{rec}(T_{in,rec} + T_{rec})$$

So $a = P_c - bk_B BT_c$ $Bk_B b = \dfrac{P_h - P_c}{T_h - T_c}$

Then $T_x = T_c + \dfrac{(Y_x - 1)}{(Y_h - 1)}(T_h - T_c)$, where $Y_x = \dfrac{P_x}{P_c}$, $Y_h = \dfrac{P_h}{P_c}$

- Not-so-simple case (unmatched, asymmetric) Three complications:
  - $p_{del} = Mp_{avail}$
  - p_{del}(to rad.) = $\eta\, p_{del}$(from source),
    and $\eta_x \neq \eta_h \neq \eta_c$
  - $a, b = a(\Gamma), b(\Gamma)$

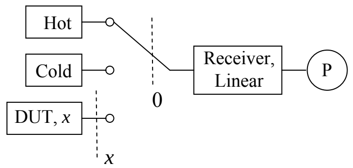
  - Handle first two by measuring and correcting.

- For dependence of a and b on Γ, have three choices:
  - tune so that $\Gamma_h = \Gamma_c = \Gamma_x$ (very narrow frequency range, need special standards)
  - characterize dependence on Γ (broadband, but a lot of work, and difficult to get good accuracy)
  - isolate (easy, accurate, but limits frequency range & difficult at low frequency)

– If isolate, *a* and *b* are (almost) independent of the source, and

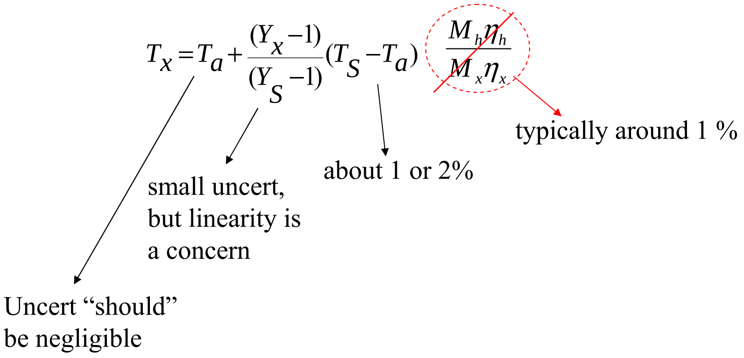
$$T_x = T_{amb} + \left(\frac{M_S \eta_S}{M_x \eta_x}\right) \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_{amb})$$

where M_x is the mismatch factor at plane x, η_x is efficiency between plane x and plane 0, etc.



Uncertainties

- Simple case (matched):



$$T_x = T_a + \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_a) \cancel{\frac{M_h \eta_h}{M_x \eta_x}}$$

typically around 1 %

about 1 or 2%

small uncert, but linearity is a concern

Uncert "should" be negligible

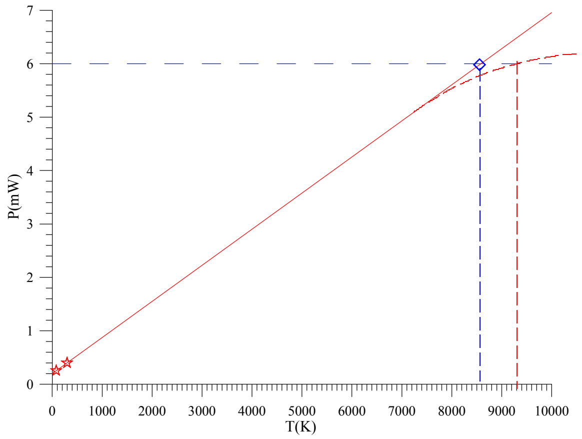
For the ENR, this ⇒ $u(\text{ENR}) \approx 0.10$ dB to 0.15 dB

- Simple-case uncerts (cont’d)
  - drift: temperature stability/control important (effect minimized by frequent switching to standards)
  - connector variability: hard to do much better than 0.1%, easy to do considerably worse.
  - Δa, Δb (due to $\Delta\Gamma$): depends on details of system, can make a crude estimate:

$T_{rev} \sim T_e$, $|\Delta\Gamma| \sim 0.05$ or 0.1

So $\Delta T_{in} \sim 0.05$ or $0.1 \times T_e$

- linearity: serious concern if T_x very different from standards, less (but some) worry if T_x near temperature of a standard.



- Uncertainties (more careful case)
  (Numbers are for NIST case) [13,14]
  - Radiometer equation:

$$T_X = T_{amb} + \frac{M_S \eta_S}{M_x \eta_x} \frac{(Y_X - 1)}{(Y_S - 1)} (T_S - T_{amb}) + (negligible)$$

  - Ambient standard:

$$\frac{u_{amb}(T_X)}{T_X} = \left| \frac{T_X - T_S}{T_a - T_S} \right| \frac{T_a}{T_X} \varepsilon_{T_a}, \quad \varepsilon_{T_a} = \frac{0.1K}{296K} = 0.034\%$$

  - “Other” standard:

$$\frac{u_{T_S}(T_X)}{T_X} = \left| 1 - \frac{T_a}{T_X} \right| \left| \frac{T_S}{T_a - T_S} \right| \frac{u(T_S)}{T_S}, \quad \frac{u(T_S)}{T_S} = 0.2\% (NIST\ W.G.), 0.8\% (NIST\ coax)$$

  - Path asymmetry: (zero if connect to same port)

$$\frac{u_{\eta/\eta}(T_X)}{T_X} = \left| 1 - \frac{T_a}{T_X} \right| u(\eta/\eta), \ u(\eta/\eta) = 0.2\% \ to \ 0.56\%$$

  - Mismatch:

$$\frac{u_{M/M}(T_X)}{T_X} = \left| 1 - \frac{T_a}{T_X} \right| u(M/M), \ u(M/M) \approx 0.2\%$$

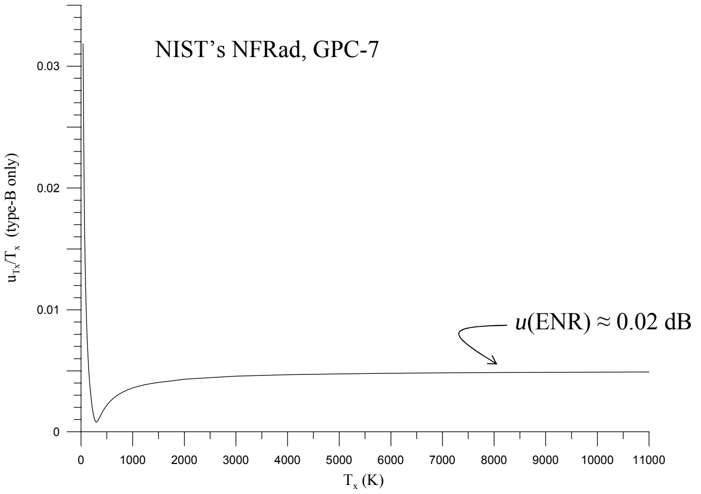
- Connectors:

$$\frac{u_{conn}(T_x)}{T_x} = u_0\left|1-\frac{T_a}{T_x}\right|\sqrt{f(GHz)}\,,\ u_0 \approx 0.053\%\ \textit{to}\ 0.069\%$$

(depending on connector type)

- Other: Nonlinearity, imperfect isolation, power ratio measurement, and broadband mismatch/frequency offset all lead to small (<0.1%) uncertainties for T_x around 10 000 K (for us/NIST).

- $u_B(T)/T$ as a function of T
  Standard relative uncertainty (1σ)

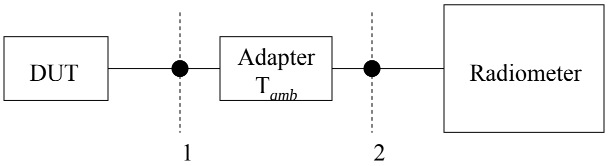


NIST's NFRad, GPC-7

u(ENR) ≈ 0.02 dB

Adapters

- Measure T at 2, want T at 1.



$$T_2 = \alpha_{21} T_{DUT} + (1-\alpha_{21}) T_{amb}$$

So $$T_{DUT} = \frac{T_2 - (1-\alpha_{21}) T_{amb}}{\alpha_{21}}$$

For a good adapter, $\alpha \approx 0.95 - 0.99$, depending on frequency.

Determine α from $\alpha_{21} = \dfrac{|S_{21}|^2 \left(1-|\Gamma_1|^2\right)}{|1-\Gamma_1 S_{11}|^2 \left(1-|\Gamma_2|^2\right)}$ or [15,16] or ….

III. NOISE FIGURE & NOISE PARAMETERS

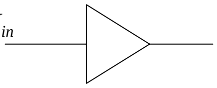
Noise Figure Defined

- Want a measure of how much noise an amplifier adds to a signal or how much it degrades the S/N ratio.

- Define Noise Figure, IEEE [17]: (at a given frequency) the ratio of total output noise power per unit bandwidth to the portion of the output noise power which is due to the input noise, evaluated for the case where the input noise power is $k_B\, T_0$, where T_0 = 290 K. (vacuum fluctuation comment)
- Noise figure & signal to noise ratio[18]:

$$\frac{(S/N)_{in}}{(S/N)_{out}} = \frac{S_{in}/290K}{GS_{in}/(G\times 290K + N_{amp})} = \frac{G\times 290K + N_{amp}}{G\times 290K} = F$$

- Effective input noise temperature:



S_{in} , N_{in}

$S_{out} = G\, S_{in}$

$N_{out} = GN_{in} + N_{amp} = Gk_B\boldsymbol{T_{in}} + \boldsymbol{N_{amp}}$

Define $N_{amp} \equiv Gk_BT_e$

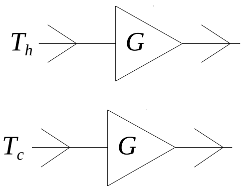
So $N_{out} = Gk_B(T_{in} + T_e)$

So Noise Figure becomes

$$F = \frac{Noise\ out}{G\times Noise\ in} = \frac{G(T_0 + T_e)}{GT_0} \qquad F(dB) = 10\log_{10}\left(\frac{T_0 + T_e}{T_0}\right)$$

Note: G, F, T_e all depend on Γ_{source}.

Simple Case Measurement, all Γ's equal



$$N_{out,h} = Gk_B(T_h + T_e)$$

$$N_{out,c} = Gk_B(T_c + T_e)$$

Combine & solve:

$$G = \frac{N_{out,h} - N_{out,c}}{k_B(T_h - T_c)} \qquad Te = \frac{N_{out,c}T_h - N_{out,h}T_c}{N_{out,h} - N_{out,c}} = \frac{T_h - YT_c}{Y-1} \quad \text{where } Y = N_{out,h}/N_{out,c}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T_h - YT_c}{(Y-1)T_0}$$

In terms of ENR:

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T_h - YT_c}{(Y-1)T_0} = \frac{ENR}{Y-1} + \left(\frac{Y}{Y-1}\right)\left(\frac{T_0 - T_c}{T_0}\right) \approx \frac{ENR_h}{(Y-1)}$$

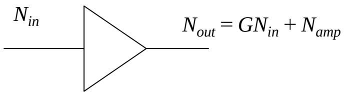
if $T_c \approx T_0$ (290 K $\Rightarrow$ ~ 63 ^{o}F), then

$$F(dB) \approx ENR_h(dB) - (Y-1)(dB)$$

Advice: Such approximations are useful in conversation or for rough estimates & mental computations. For any "real" computation, use the full, correct expression(s). It only takes a few seconds of extra typing, and it can make a difference in the answer.

Noise-Temperature Definition Revisited

- Quantum I: Equivalent black-body definition vs. “power” definition.



“Power” definition: $N = kT$,

then $N_{in} = kT_{in}$, $N_{out} = kT_{out}$, $N_{amp} = kGT_e$,

so $kT_{out} = kG(T_{in} + T_e)$

and $T_{out} = G(T_{in} + T_e)$

“Equivalent black-body temperature” definition: $N = \frac{hf}{e^{hf/kT} - 1}$

so $N_{out} = GN_{in} + N_{amp}$ becomes (after dividing by k)

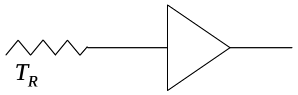
$$\frac{hf}{e^{hf/kT_{out}} - 1} = G\left(\frac{hf}{e^{hf/kT_{in}} - 1} + \frac{hf}{e^{hf/kT_e} - 1}\right).$$

Solving for T_{out} , we would get

$$T_{out} = \frac{hf}{k}\left\{\ln\left[1 + \frac{1}{G}\left(\frac{1}{\left(e^{hf/kT_{in}} - 1\right)} + \frac{1}{\left(e^{hf/kT_e} - 1\right)}\right)^{-1}\right]\right\}^{-1}$$

- Quantum II: Vacuum-fluctuation contribution
  - Continual “sea” of virtual particle-antiparticle pairs everywhere.
  - Cannot extract energy from them (from the vacuum), but they can effect physical processes; & in particular they add noise to active electronic devices [19 – 21].
  - They result in an additional effective input noise temperature of $hf/2k_B$ at the inut of an amplifier.
  - This is very small, *usually* negligible at microwave freqeuncies, $T_{vac} = 0.24$ K at 10 GHz, but it is there, & there are some cases where it is not negligible [22].
  - It results in a minimum output noise from an amplifier, $N_{out,min} = Ghf/2$.

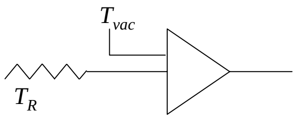
- Not yet a general agreement on how to include T_{vac} in definition of noise temperatures.
- Can include it in T_e (blame it on the amp)



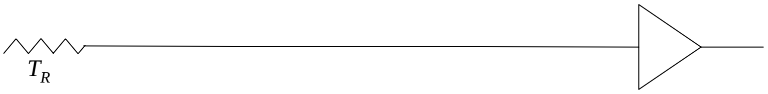
$$T_{out} = G(\underbrace{T_R + \rlap{$\overbrace{\phantom{T_{vac} + T_e}}^{T_e'}$}T_{vac}}_{T_{in}} + T_e)$$

  or can include it in T_{in} .
- We’ll include it in T_{in} [7, 22].
- Also a question of whether to include T_{vac} as part of the source T_R (as in [7]) or as a separate input source [22].

– I prefer keeping it as a separate input [22].



– One reason: case of large separation distance (especially in remote sensing, for example)

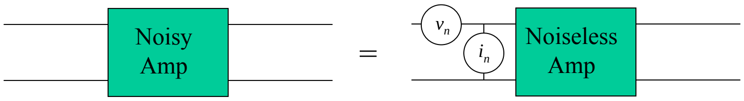


– Note: get same/consistent results, independent of which way you group things.

Noise Parameters, IEEE Representation

- Simple case was $T_e = \frac{T_h - YT_c}{Y-1}$, $Y = \frac{N_{out,h}}{N_{out,c}}$
- But that’s just for one value of Γ_{source}. Want to determine F or T_e for any Γ_{source}. So parameterize dependence on Γ_{source}.
- Several parameterizations in use; most common are variants of the IEEE [23] form.

- Equivalent circuit:



- (Noise out)/(Noise in) depends on impedance of input termination, $NF = NF(Z_S)$ or $NF(\Gamma_S)$, & $T_e = T_e(Z_S$ or $\Gamma_S)$,

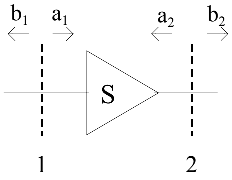
$$NF = NF_{\min} + \frac{4R_n}{Z_0}\frac{\left|\Gamma_{opt} - \Gamma_S\right|^2}{\left|1+\Gamma_{opt}\right|^2\left(1-\left|\Gamma_S\right|^2\right)} \qquad T_e = T_{e,\min} + t\frac{\left|\Gamma_{opt} - \Gamma_S\right|^2}{\left|1+\Gamma_{opt}\right|^2\left(1-\left|\Gamma_S\right|^2\right)}$$

4 parameters: $T_{e,min}$, $t = 4R_nT_0/Z_0$, and complex Γ_{opt} .

Note: many equivalent forms of IEEE representation; this one is from [24].

Wave Representation of Noise Matrix

- For microwave radiometry, wave representation [24 – 29] provides more flexibility.
- Linear 2-port:



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

- Noise correlation matrix is defined by

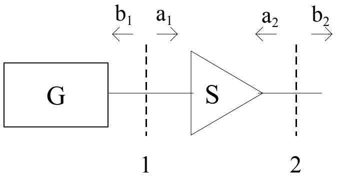
$$N_{ij} = \langle b_i b_j^* \rangle$$

or $\hat{N}_{ij} = \langle \hat{b}_i \hat{b}_j^* \rangle$ for intrinsic noise matrix

- Four real noise parameters:

$$\left\langle \left|\hat{b}_1\right|^2 \right\rangle, \left\langle \left|\hat{b}_2\right|^2 \right\rangle, \left\langle \hat{b}_1 \hat{b}_2^* \right\rangle$$

- Output noise temperature T_2



$$k_B T_2 = \frac{|S_{21}|^2}{\left(1-|\Gamma_{GS}|^2\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$N_G = \frac{\left(1-|\Gamma_G|^2\right)}{|1-\Gamma_G S_{11}|^2} k_B T_G$$

$$N_1 = \left|\frac{\Gamma_G}{1-\Gamma_G S_{11}}\right|^2 \left\langle \left|\hat{b}_1\right|^2 \right\rangle$$

$$N_2 = \left\langle \left|\hat{b}_2 / S_{21}\right|^2 \right\rangle$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{\Gamma_G}{(1-\Gamma_G S_{11})} \left\langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \right\rangle\right]$$

- So for T_e we have

$$T_e = \frac{|\Gamma_G|^2}{(1-|\Gamma_G|^2)} X_1 + \frac{|1-\Gamma_G S_{11}|^2}{(1-|\Gamma_G|^2)} X_2 + \frac{2}{(1-|\Gamma_G|^2)} \mathrm{Re}\left[(1-\Gamma_G S_{11})^* \Gamma_G X_{12}\right]$$

where $k_B X_1 \equiv \left\langle |\hat{b}_1|^2 \right\rangle$, $k_B X_2 \equiv \left\langle |\hat{b}_2 / S_{21}|^2 \right\rangle$, $k_B X_{12} \equiv \left\langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \right\rangle$

- Whereas IEEE parameterization is

$$T_e = T_{e,\min} + t \frac{|\Gamma_G - \Gamma_{opt}|^2}{\left(1-|\Gamma_G|^2\right)|1+\Gamma_{opt}|^2}$$

- Can relate the two:

<u>X’s → IEEE</u>

$$t = X_1 + |1+S_{11}|^2 X_2 - 2\,\mathrm{Re}\left[(1+S_{11})^* X_{12}\right],$$

$$T_{e,\min} = \frac{X_2 - |\Gamma_{opt}|^2 \left[X_1 + |S_{11}|^2 X_2 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)\right]}{\left(1+|\Gamma_{opt}|^2\right)},$$

$$\Gamma_{opt} = \frac{\eta}{2}\left(1 - \sqrt{1 - \frac{4}{|\eta|^2}}\right),$$

$$\eta = \frac{X_2\left(1+|S_{11}|^2\right) + X_1 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)}{(X_2 S_{11} - X_{12})}.$$

<u>IEEE → X’s</u>

$$X_1 = T_{e,\min}\left(|S_{11}|^2 - 1\right) + \frac{t|1 - S_{11}\Gamma_{opt}|^2}{|1+\Gamma_{opt}|^2},$$

$$X_2 = T_{e,\min} + \frac{t|\Gamma_{opt}|^2}{|1+\Gamma_{opt}|^2},$$

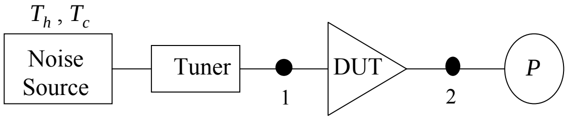
$$X_{12} = S_{11} T_{e,\min} - \frac{t\Gamma_{opt}^* \left(1 - S_{11}\Gamma_{opt}\right)}{|1+\Gamma_{opt}|^2}.$$

Notes:
$X_2 = T_{e,0}$
Bound implied by $X_1 \geq 0$

Measuring Noise Parameters

- Many different methods [24, 26, 28, 30 – 41], most based on IEEE parameterization.
- Basic idea of (almost) all methods is to
  - present amplifier (or device) with a variety of different known input terminations (Γ & T),
  - have an equation for the "output" in terms of the noise parameters and known quantities (Γ's, T's, S-parameters),
  - determine noise parameters by a fit to the measured output.
  - Need good distrib. of Γ's in complex plane.

- Can fit for noise figure [28]

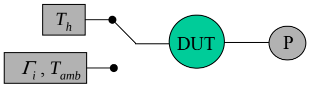


$$NF = NF_{\min} + \frac{4R_n}{Z_0} \frac{\left|\Gamma_{opt} - \Gamma_1\right|^2}{\left|1 + \Gamma_{opt}\right|^2 \left(1 - \left|\Gamma_1\right|^2\right)}$$

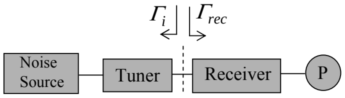
Notes:

- Use tuner to get different Γ_1, measure with T_h and T_c for each Γ_1 to get NF for that Γ_1.
- Must correct for tuner to get T_{in} at 1. Must calibrate receiver for each value of Γ_2 (or have isolator in front of receiver).

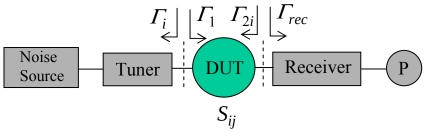
- Or can fit for output power [31, 42, 43]. This is the most popular method now.



In practice, first measure noise parameters of receiver,

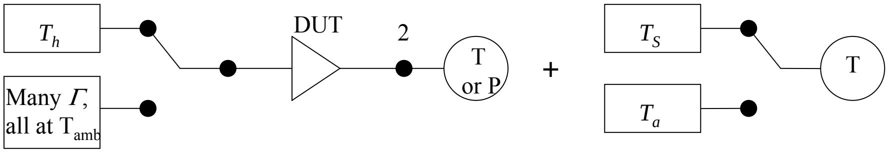


Then measure DUT + receiver



and extract DUT noise parameters.

- Noise-matrix approach [28, 29, 38, 44] to measuring noise parameters:



$$k_B T_2 = \frac{|S_{21}|^2}{\left(1-|\Gamma_{GS}|^2\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$\Gamma_{GS} = S_{22} + \frac{\Gamma_G S_{12} S_{21}}{(1-\Gamma_G S_{11})}$$

$$N_G = \frac{\left(1-|\Gamma_G|^2\right)}{|1-\Gamma_G S_{11}|^2} k_B T_G$$

$$N_1 = \left|\frac{\Gamma_G}{1-\Gamma_G S_{11}}\right|^2 k_B X_1$$

$$N_2 = k_B X_2$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{\Gamma_G}{(1-\Gamma_G S_{11})} k_B X_{12}\right]$$

- Noise-Parameter Uncertainties
  - Monte Carlo method is probably the most practical [33, 44 – 47]
  - Some general approximate features [44]:
    - Uncerts in *G* and $T_{\min}$ (& $F_{\min}$) are dominated by uncert in T_h. 0.1 dB uncert in T_h → ~ 0.1 dB uncert in *G* and $F_{\min}$.
    - Uncerts in Γ_{opt} are dominated by uncerts in Γ_G's. Uncert in Re or Im Γ_{opt} is ~ 3 or 4× uncert in Re or Im Γ_G (for 13 terminations).
    - *t* (or R_n) is sensitive to just about everything.
    - T_{amb} is not a major factor, because it is known much better than T_h. Note, however, that it could affect T_h or the amplifier properties.

Measuring Noise Parameters on Wafer

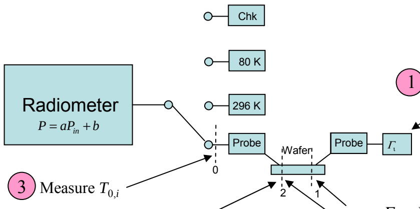
- Just like amplifier noise parameters—only harder.
- Harder due to probes and to device properties.
- Complications due to Probes:
  - Must characterize probes: on-wafer standards ⇒ larger uncertainties for Γ's, *S*-parameters, T_{in}, T_{out}.
  - Restricted range of Γ's (due to loss in probe).
  - Potential contact problems, vibrations.

- Complications due to Device:
  - If measuring an on-wafer amplifier, no additional device-related problems (assuming it’s well matched).

  But for a transistor:

  - Matching problems, large S_{11}, $S_{22} \Rightarrow$ larger corrections & therefore larger uncertainties.
  - Large Γ_{opt} , near edge of Smith chart.
  - Smaller noise figures/noise temps than amps.

- Procedure used at NIST [48, 49]:



① Series of different terminations i

② $\Gamma_{1,i}$, $\Gamma_{2,i}$, and S_{ij} measured with on-wafer cal, $T_{1,i}$ measured on wafer

③ Measure $T_{0,i}$

④ Compute $T_{2,i}$: $T_0 = \alpha_{02} T_2 + (1-\alpha_{02}) T_{amb}$ so $T_{2,i} = \dfrac{T_{0,i} - (1-\alpha_{02,i}) T_{amb}}{\alpha_{02,i}}$

$$\text{where} \quad \alpha_{02.i} \equiv \frac{P_0^{avail}}{P_2^{avail}} = \frac{|S_{21}(02)|^2 \left(1 - |\Gamma_{2,i}|^2\right)}{|1 - \Gamma_{2,i} S_{11}(02)|^2 \left(1 - |\Gamma_{0,i}|^2\right)}$$

⑤ Measure T_{rev} = T_1 in reverse configuration

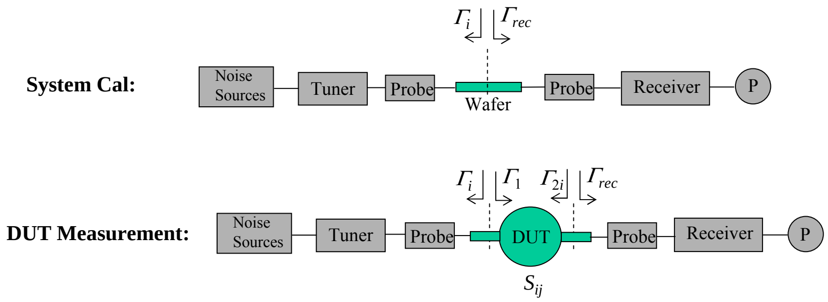
⑥ (Weighted) Fit to

$$T_{2,i} = \frac{|S_{21}|^2}{\left(1 - |\Gamma_{2,i}|^2\right)} \left\{ \frac{\left(1 - |\Gamma_{1,i}|^2\right)}{|1 - \Gamma_{1,i} S_{11}|^2} T_{1,i} + \left| \frac{\Gamma_{1,i}}{1 - \Gamma_{1,i} S_{11}} \right|^2 X_1 + X_2 + 2\,\mathrm{Re}\left[\frac{\Gamma_{1,i} X_{12}}{1 - \Gamma_{1,i} S_{11}} \right] \right\}$$

and

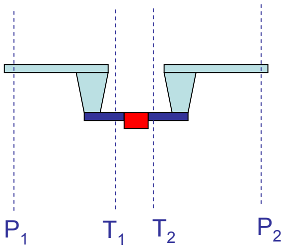
$$T_1(rev.config) = \frac{1}{\left(1 - |\Gamma_1|^2\right)} \left\{ \frac{|S_{12}|^2 \left(1 - |\Gamma_{2,L}|^2\right)}{|1 - \Gamma_{2,L} S_{22}|^2} T_{2,L} + \left| \frac{S_{12} S_{21} \Gamma_{2,L}}{1 - \Gamma_{2,L} S_{22}} \right|^2 X_2 + X_1 + 2\,\mathrm{Re}\left[\frac{S_{12} S_{21} \Gamma_{2,L} X_{12}{}^*}{1 - \Gamma_{2,L} S_{22}} \right] \right\}$$

- Commercial Systems [e.g., 42, 43]: similar to general noise parameters (above), except that reference planes are on wafer.

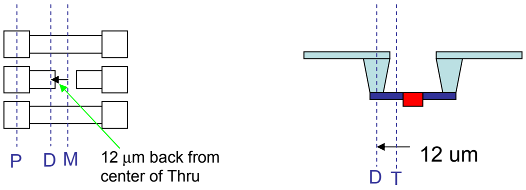


- Must therefore calibrate at those reference planes on wafer. Commonly done at probe tip, with an "off-wafer" cal set.

- To get properties of device itself, must remove effects of lines between the calibration reference planes (P_1 and P_2) and the device reference planes (T_1 and T_2). To do so, measure auxiliary standards (short, open) between planes T_1 and T_2, and "deembed" [50].



- NIST on-wafer calibration (Statistical) calibrates at center of through (M) and translates back (to D). Would still need to “deembed” to get down to T.



IV. NOISE-PARAMETER CHECKS & VERIFICATION

- So how do we convince ourselves that our noise-parameter measurement results might be correct?
- Will give three tests:
  - – measure noise parameters of passive device, such as attenuator
  - – measure T_{rev}
  - – Cascade test

Attenuator Test

- Noise matrix of a passive device (such as an attenuator) is given by Bosma's theorem,

$$\left\langle \hat{b}_i \hat{b}_j^* \right\rangle = kT\left(\boldsymbol{1} - \boldsymbol{SS}^+\right)_{ij}$$

- So for an attenuator at (noise) temperature T_a,

$$X_1 = \left(1 - |S_{11}|^2 - |S_{12}|^2\right)T_a$$

$$X_2 = \frac{\left(1 - |S_{22}|^2 - |S_{21}|^2\right)}{|S_{21}|^2}T_a$$

$$X_{12} = -\frac{\left(S_{21}^* S_{11} + S_{12} S_{22}^*\right)}{S_{21}^*}T_a$$

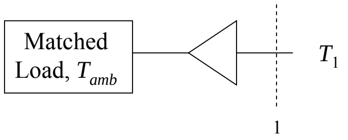
- So, measure noise parameters of an attenuator & see if you get the correct answers.

- Other passive devices as tests (especially on a wafer):
  - Cold FET [51]
  - Lange Coupler [52]

  These have the advantage of being poorly matched, & therefore more similar to the devices of interest.

T_{rev} Test [34, 38, 53]

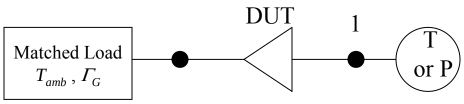
- T_{rev} test: Measure noise temp from input of amplifier, when output is terminated in a matched load.



- Can show that for $\Gamma_L S_{21} S_{12}$ small,

$$T_{rev} \approx \frac{X_1}{\left(1-|\Gamma_1|^2\right)}$$

- Full form is:



$$T_1 = \frac{1}{\left(1-|\Gamma_1|\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$\Gamma_1 = S_{11} + \frac{\Gamma_G S_{12} S_{21}}{(1-\Gamma_G S_{22})}$$

$$N_G = \frac{|S_{12}|^2\left(1-|\Gamma_G|^2\right)}{|1-\Gamma_G S_{22}|^2} k_B T_{amb}$$

$$N_1 = k_B X_1$$

$$N_2 = \left|\frac{S_{12} S_{21} \Gamma_G}{1-\Gamma_G S_{22}}\right|^2 k_B X_2$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{S_{12} S_{21} \Gamma_G}{(1-\Gamma_G S_{22})} k_B X_{12}^*\right]$$

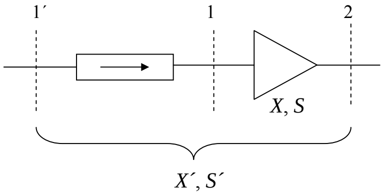
- So measure T_{rev}, compare to value predicted from the value of X_1 from the noise-parameter determination.
- If working in terms of IEEE parameters, convert, using

$$X_1 = T_{e,\min}\left(|S_{11}|^2 - 1\right) + \frac{t|1 - S_{11}\Gamma_{opt}|^2}{|1+\Gamma_{opt}|^2} \;,$$

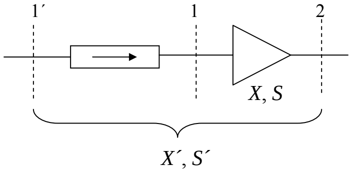
$$X_2 = T_{e,\min} + \frac{t|\Gamma_{opt}|^2}{|1+\Gamma_{opt}|^2} \;,$$

$$X_{12} = S_{11}T_{e,\min} - \frac{t\Gamma_{opt}^*\left(1 - S_{11}\Gamma_{opt}\right)}{|1+\Gamma_{opt}|^2} \;.$$

Cascade Test [53]



- Connect an isolator (or other passive 2-port) to amplifier input & measure noise parameters of combination.
- *X*´ parameters can be written in terms of *X* parameters (amp alone) and the *S*-parameters of amp and isolator.
- Using Bosma’s theorem and standard *S*-parameter algebra, can show



$$X_1' = \left|\frac{S_{12}^I}{1 - S_{11}S_{22}^I}\right|^2 X_1 + T_I(A_1 - A_2),$$

$$A_1 = \left\{\left(1 - \left|S_{11}^I\right|^2 - \left|S_{12}^I\right|^2\right) + \left|\frac{S_{11}S_{12}^I}{1 - S_{11}S_{22}^I}\right|^2 \left(1 - \left|S_{21}^I\right|^2 - \left|S_{22}^I\right|^2\right)\right\},$$

$$A_2 = 2\,\mathrm{Re}\left[\frac{S_{12}^I S_{11}}{(1 - S_{11}S_{22}^I)}\left(S_{21}^I S_{11}^{I*} + S_{12}^{I*} S_{22}^I\right)\right],$$

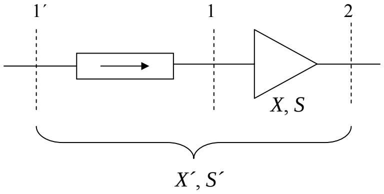
$$X_2' = \frac{1}{\left|S_{21}^I\right|^2}\left\{\left|1 - S_{11}S_{22}^I\right|^2 X_2 + \left|S_{22}^I\right|^2 X_1 + 2\,\mathrm{Re}\left[S_{22}^I\left(1 - S_{11}S_{22}^I\right)^* X_{12}\right] + T_I\left(1 - \left|S_{22}^I\right|^2 - \left|S_{21}^I\right|^2\right)\right\},$$

$$X_{12}' = \frac{S_{12}^I\left(1 - S_{11}S_{22}^I\right)^*}{S_{21}^{I*}\left(1 - S_{11}S_{22}^I\right)} X_{12} + \frac{S_{12}^I S_{22}^{I*}}{S_{21}^{I*}\left(1 - S_{11}S_{22}^I\right)} X_1 - T_I A_3,$$

$$A_3 = \left[\left(\frac{S_{21}^{I*}S_{11}^I + S_{12}^I S_{22}^{I*}}{S_{21}^{I*}}\right) - \frac{S_{12}^I S_{11}}{S_{21}^{I*}\left(1 - S_{11}S_{22}^I\right)}\left(1 - \left|S_{22}^I\right|^2 - \left|S_{21}^I\right|^2\right)\right],$$

Note: could instead use an attenuator (for on wafer).

- Approximate expressions (for isolator case):



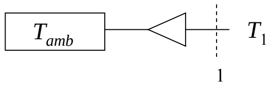
$$X_1' \approx T_I,$$

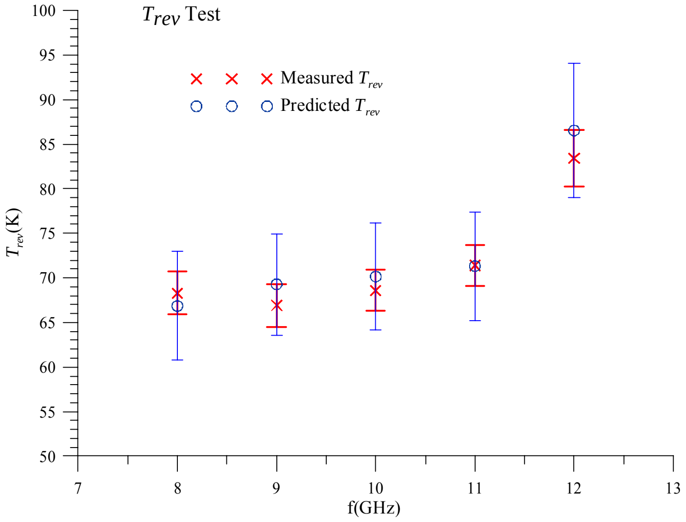
$$X_2' \approx \frac{\left(X_2 + T_I\left(1 - \left|S_{21}^I\right|^2\right)\right)}{\left|S_{21}^I\right|^2},$$

$$X_{12}' \approx \frac{S_{12}^I}{S_{21}^{I*}} X_{12} - T_I S_{11}^I,$$

X_{12}' is small and (approximately) independent of amplifier; excellent verification test.

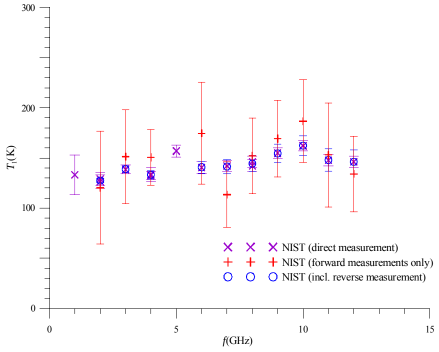
Samples of Test Results [53]

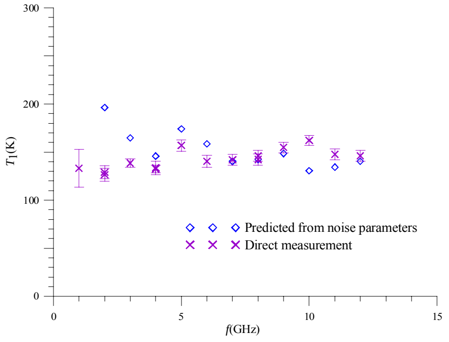




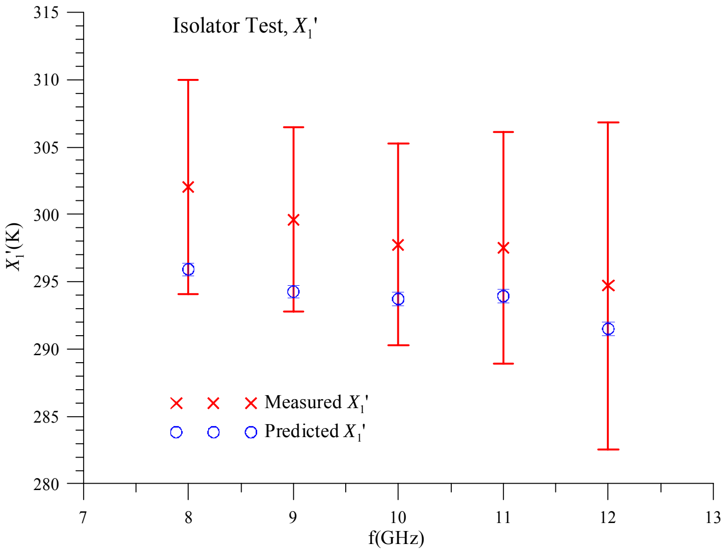
Error bars are standard uncertainties (1σ).

T_{rev} test on wafer [54]

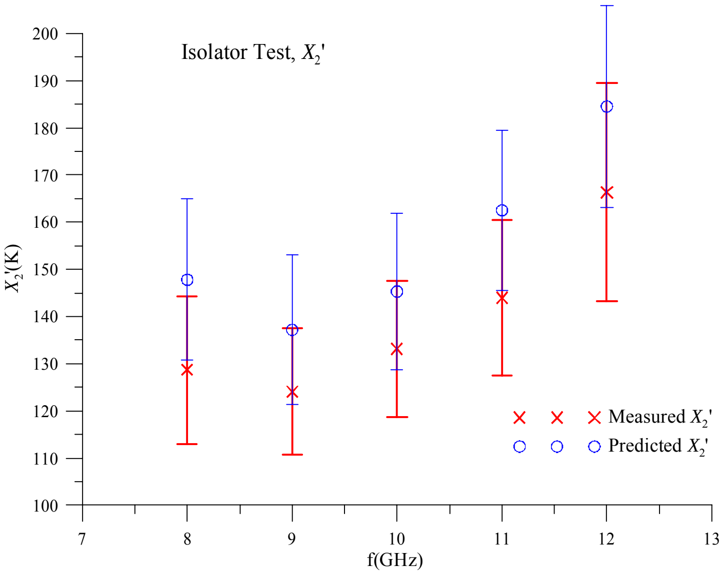




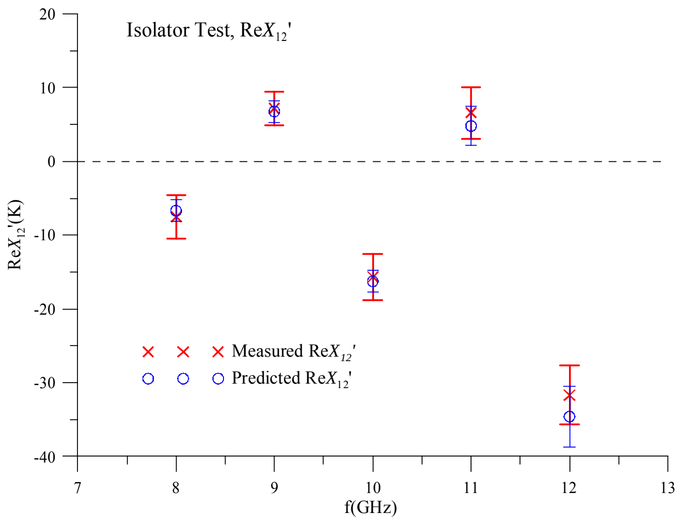
Error bars are standard uncertainties (1σ).



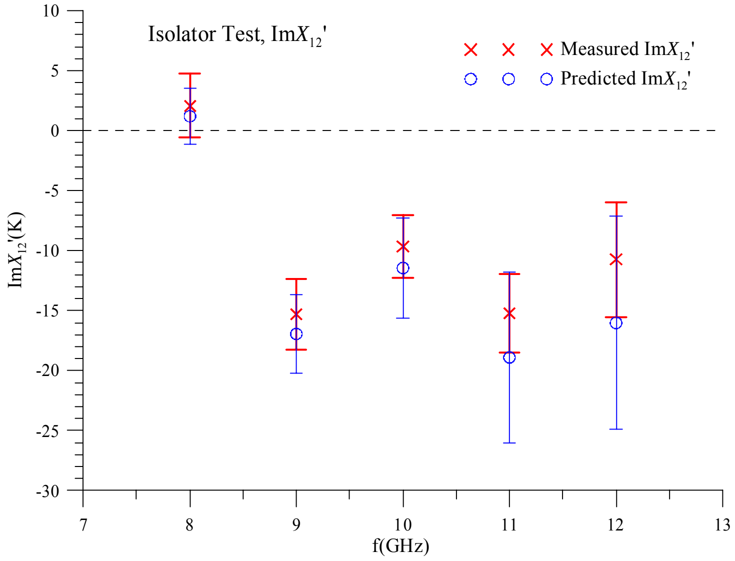
Error bars are standard uncertainties (1σ).



Error bars are standard uncertainties (1σ).



Error bars are standard uncertainties (1σ).



Error bars are standard uncertainties (1σ).

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http://boulder.nist.gov/div818/81801/Noise/index.html

REFERENCES

(NIST references available at http://boulder.nist.gov/div818/81801/Noise/publications/noise_pubs.html)

[1] H. Nyquist, Phys. Rev., vol. 32, pp. 110 – 113 (1928).

[2] A. van der Ziel, *Noise*. Prentice-Hall, NY: 1954.

[3] W.B. Davenport and W.L. Root, *Random Signals and Noise*. McGraw-Hill, NY: 1958.

[4] F. Rief, *Fundamentals of Statistical and Thermal Physics*, Ch 15 (especially Sections 8 and 13 – 16). McGraw-Hill, NY: 1965.

[5] IEEE Standard Dictionary of Electrical and Electronic Terms, Fourth Edition, 1988.

[6] J. Randa *et al.*, “International comparison of thermal noise-temperature measurements at 2, 4, and 12 GHz,” IEEE Trans. on Instrum. and Meas., vol. 48, no. 2 pp. 174 – 177 (1999).

[7] A.R. Kerr, “Suggestions for revised definitions of noise quantities, including quantum effects, IEEE Trans. Microwave Theory and Tech., vol. 47, no. 3, pp. 325 – 329, March 1999.

[8] D.M. Kerns and R.W. Beatty, *Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis*. Pergamon Press, London: 1974.

[9] for my notation and conventions, see: J. Randa, “Noise temperature measurements on wafer,” NIST Tech. Note 1390, March 1997.

[10] N. Skou, *Microwave Radiometer Systems: Design and Analysis*. Artech House, Norwood, MA: 1989.

[11] W.C. Daywitt, “Radiometer equation and analysis of systematic errors for the NIST automated radiometers,” NIST Tech. Note 1327, March 1989.

[12] J. Randa and L.A. Terrell, “Noise-temperature measurement system for the WR-28 band,” NIST Tech. Note 1395, Aug. 1997.

[13] J. Randa, “Uncertainties in NIST noise-temperature measurements,” NIST Tech. Note 1502, March 1998.

[14] C. Grosvenor, J. Randa, and R.L. Billinger, “Design and testing of NFRad—a new noise measurement system,” NIST Tech. Note 1518, April 2000.

[15] S. Pucic and W.C. Daywitt, “Single-port technique for adapter efficiency evaluation,” *45th ARFTG Conference Digest*, pp. 113 – 118, Orlando, FL, May 1995.

[16] J. Randa, W. Wiatr, and R.L. Billinger, “Comparison of adapter characterization methods,” IEEE Trans. Microwave Theory and Tech., vol.47, no. 12, pp. 2613 – 2620, December 1999.

[17] H.A. Haus *et al*., “IRE standards on methods of measuring noise in linear twoports, 1959,” Proc. IRE, vol. 48, no. 1, pp. 60 – 68, January 1960.

[18] H.T. Friis, “Noise figures of radio receivers,” Proc. IRE, 419 – 422, July 1944.

---

[19] H.B. Callen and T.A. Welton, “Irreversibility and generalized noise,” Phys. Rev. vol. 83, no. 1, pp. 34 – 40, July 1951.

[20] C.M. Caves, “Quantum limits on noise in linear amplifiers,” Phys. Rev. D, vol 26, no. 8, pp. 1817 – 1839, Oct. 1982.

[21] J.R. Tucker and M.J. Feldman, “Quantum detection at millimeter wavelengths,” Rev. Mod. Phys., vol. 57, no. 4, pp. 1055 – 1113, Oct. 1985.

[22] J. Randa, E. Gerecht, D. Gu, and R.L. Billinger, “Precision measurement method for cryogenic amplifier noise temperatures below 5 K,” IEEE Trans. Microwave Theory and Tech., vol. 54, no. 3, pp. 1180 – 1189, March 2006.

[23] H.A. Haus *et al*., “Representation of noise in linear twoports,” Proc. IRE, vol. 48, no. 1, pp. 69 – 74, January 1960.

[24] R.P. Meys, “A wave approach to the noise properties of linear microwave devices,” IEEE Trans. Microwave Theory and Tech., vol. MTT-26, pp. 34 – 37, January 1978.

[25] D.F. Wait, “Thermal noise from a passive linear multiport,” IEEE Trans. Microwave Theory and Tech., vol. MTT-16, no. 9, pp. 687 – 691, September 1968.

[26] R.P. Hecken, “Analysis of linear noisy two-ports using scattering waves,” IEEE Trans. Microwave Theory and Tech., vol.MTT-29, no. 10, pp. 997 – 1004, October 1981.

[27] S.W. Wedge and D.B. Rutledge, “Noise waves and passive linear multiports,” IEEE Microwave and Guided Wave Letters, vol. 1, no. 5, pp 117 – 119, May 1991.

[28] S.W. Wedge and D.B. Rutledge, “Wave techniques for noise modeling and measurement,” IEEE Trans. Microwave Theory and Tech., vol.40, no. 11, pp. 2004 – 2012, November 1992.

[29] for my notation: J. Randa, “Noise characterization of multiport amplifiers,” IEEE Trans. Microwave Theory and Tech., vol. 49, no. 10, pp. 1757 – 1763, October 2001.

[30] R.Q. Lane, “The determination of device noise parameters,” Proc. IEEE, Vol. 57 pp. 1461 – 1462, August 1969.

[31] V. Adamian and A. Uhlir, “A novel procedure for receiver noise characterization,” IEEE Trans. Instrum. and Meas., vol. IM-22, pp. 181 – 182, June 1973.

[32] M.W. Pospieszalski, “On the measurement of noise parameters of microwave two-ports,” IEEE Trans. Microwave Theory and Tech., vol. MTT-34, no. 4, pp. 456 – 458, April 1986.

[33] A.C. Davidson, B.W. Leake, and E. Strid, “Accuracy improvements in microwave noise parameter measurements,” IEEE Trans. Microwave Theory and Tech., vol. 37, no. 12, pp. 1973 – 1978, December 1989.

[34] D.F. Wait and G.F. Engen, “Application of radiometry to the accurate measurement of amplifier noise,” IEEE Trans. Instrum. and Meas., vol. 40, no. 2, pp. 433 – 437, April 1991.

[35] A. Boudiaf and M. Laporte, “An accurate and repeatable technique for noise parameter measurements, “IEEE Trans. Instrum. and Meas., vol. 42, no. 2, pp. 532 – 537, April 1993.

[36] G. Martines and M. Sannino, “The determination of the noise, gain and scattering parameters of microwave transistors …”, IEEE Trans. Microwave Theory and Tech., vol. 42, no. 7, pp. 1105 – 1113, July 1994.

[37] G.L. Williams, “Measuring amplifier noise on a noise source calibration radiometer,” IEEE Trans. Instrum. and Meas., vol. 44, no. 2, pp. 340 – 342, April 1995.

[38] D.F. Wait and J. Randa, “Amplifier noise measurements at NIST,” IEEE Trans. Instrum. and Meas., vol. 46, no. 2, pp. 482 – 485, April 1997.

[39] T. Werling, E. Bourdel, D. Pasquet, and A. Boudiaf, “Determination of wave noise sources using spectral parametric modeling,” IEEE Trans. Microwave Theory and Tech., vol. 45, no. 12, pp. 2461 – 2467, December 1997.

[40] A. Lazaro, L. Pradell, and J.M. O’Callaghan, “FET noise-parameter determination using a novel technique based on 50-Ω noise-figure measurements,” IEEE Trans. Microwave Theory and Tech., vol. 47, no. 3, pp. 315 – 324, March 1999.

[41] Your favorite method, which I’ve probably omitted (inadvertently, honest).

[42] V. Adamian, "2 – 26.5 GHz on-wafer noise and S-parameter measurements using a solid state tuner," *34th ARFTG Conference Digest, pp. 33 – 40*, Ft. Lauderdale, FL, Nov. 1989.

[43] C.E. Woodin, D.L. Wandrei, and V. Adamian, "Accuracy improvements to on-wafer amplifier noise figure measurements," *38th ARFTG Conference Digest*, pp. 129 – 138, San Diego, CA, December 1991.

[44] J. Randa, "Noise-parameter uncertainties: a Monte Carlo simulation," J. Res. Natl. Stand. Technol., vol. 107, pp. 431 – 444, 2002.

[45] V. Adamian, "Verification and accuracy of noise parameter measurements," Unpublished lecture notes, 1991.

[46] M.L. Schmatz, H.R. Benedickter, and W. Bächtold, "Accuracy improvements in microwave noise parameter determination," *Digest of the 51st ARFTG Conference*, Baltimore, MD, June 1998.

[47] S. Van den Bosch and L. Martens, "Improved impedance-pattern generation for automatic noise-parameter determination," IEEE Trans. Microwave Theory and Tech., vol 46, no. 11, pp. 1673 – 1678, November 1998.

[48] J. Randa *et al.*, "Interlaboratory comparison of noise-parameter measurements on CMOS devices with 0.12 µm gate length," *66th ARFTG Microwave Measurements Conference Digest*, pp. 77 – 81, Washington, DC, December 2005.

[49] J. Randa and D.K Walker, "On-wafer measurement of transistor noise parameters at NIST," IEEE Trans. I&M, to be published, 2007.

[50] C.-H. Chen and M.J. Deen, "RF CMOS noise characterization and modeling," in *CMOS RF Modeling, Characterization, and Applications*, ed. M.J. Deen and T.A. Fjeldly, World Scientific, River Edge, NJ, 2002.

[51] L. Escotte, R. Plana, J. Rayssac, O. Llopis, and J. Graffeuil, "Using cold FET to check accuracy of microwave noise parameter test set," Electronics Letters, vol. 27, no 10, pp. 833 – 835, May 1991.

[52] A. Boudiaf, C. Dubon-Chevallier, and D. Pasquet, "Verification of on-wafer noise parameter measurements," IEEE Trans. Instrum. and Meas., vol. 44, no. 2, pp. 332 – 335, April 1995.

[53] J. Randa and D.K. Walker, "Amplifier noise-parameter measurement checks and verification, *Digest of the 63rd ARFTGMicrowave Measurments Conference*, pp. 41 – 45, Ft. Worth, TX, June 2004.

[54] J. Randa & *al.*, "Reverse noise measurement and use in device characterization," *2006 IEEE Radio Frequency Integrated Circuits (RFIC) Symposium Digest*, pp. 345 – 348, San Francisco, June 2006.

NIST references available at http://boulder.nist.gov/div818/81801/Noise/publications/noise_pubs.html